

Note on a Pattern from CP Violation in Neutrino Oscillations

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October 14, 2010

Abstract

A virtual graphical construction is made to show the difference between neutrino and anti-neutrino oscillations in the presence of CP violation with CPT conservation.

1 Introduction

There is interest in the possibility that CPT violation may occur and then show in neutrino oscillation experiments[1],[2], [3].. However this may be, CP violation is long established and it is of importance to seek it in neutrino oscillation results. If one takes a conservative point of view that nature conserves CPT, and there are 3 generations of neutrinos, then the consequences of CP violation in neutrino oscillation become more definite. In particular if CP were conserved ν transition probabilities would be the same as $\bar{\nu}$ transition probabilities while the occurrence therein of CP violation makes these different [4]. This difference is dependent on the neutrino oscillation parameter L/E , L being the distance of travel from creation to detection and E the energy of the initial neutrino: and it is also sensitive to the value of the small ratio of neutrino mass squared differences. There results a complicated 2-variable dependence in addition to the linear dependence on the leptonic Jarlskog parameter, J_{lep} [5].

2 Pattern for the difference between ν and $\bar{\nu}$ oscillations

The input to the formula for neutrino transition probabilities is largely from the mixing matrix elements $U_{\alpha i}$ where α is one of the 3 flavour indices and i one of the 3 mass eigenstate indices. From these 9 elements plaquettes [6] can be constructed, these being phase invariant products of 2 U elements multiplied by products of 2 U^* elements which occur in transition probabilities $\nu_\alpha \rightarrow \nu_\beta$ of neutrino beams. Here α, β are flavour indices of beam neutrinos. The construction is as follows.

Greek letters denoting flavour indices (e, μ, τ) and Roman letters mass eigenstate indices there are 9 plaquettes, labelled $\Pi_{\alpha i}$:

$$\Pi_{\alpha i} \equiv U_{\beta j} U_{\beta k}^* U_{\gamma k} U_{\gamma j}^* \quad (1)$$

where α, β, γ are non-equal and in cyclic order and i, j, k are also non-equal and in cyclic order (The pattern discussed in this paper applies for an inverted hierarchy as well as for the normal hierarchy; that is there is no necessary association between a particular α and a particular i .) .

Making use of well-known formalism [4] the beam transition probability for $\nu_\alpha \rightarrow \nu_\beta, \alpha \neq \beta$ can be written as

$$P(\nu_\alpha \rightarrow \nu_\beta) = -4 \sum_{i=1}^3 \Re(\Pi_{\gamma i}) \sin^2((m_{\nu k}^2 - m_{\nu j}^2)L/4E) \quad (2)$$

$$+ 2 \sum_{i=1}^3 \Im(\Pi_{\gamma i}) \sin((m_{\nu k}^2 - m_{\nu j}^2)L/2E) \quad (3)$$

where L is the length travelled by neutrino energy E from creation to annihilation at detection. The survival probability, $P(\nu_\alpha \rightarrow \nu_\alpha)$, (given in [4]) can be calculated from the transition probabilities above. So the 3×3 plaquette matrix Π and the neutrino mass eigenstate values squared differences carry all the information on transition and survival probabilities of a given beam.

The last term (3) being only non-zero when CP is not conserved. Indeed all the nine $\Im \Pi_{\alpha i}$ are equal and equal to [7] the leptonic Jarlskog invariant;

$$\Im \Pi_{\alpha i} = J_{lep}$$

. So J , as usual, is signalling CP violation. With CPT invariance the transition probability for anti-neutrinos $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta; \Pi) = P(\nu_\alpha \rightarrow \nu_\beta; \Pi^*)$ [4]. Thus the contribution of CP violation in anti-neutrino transitions is of the same magnitude but opposite sign to that in neutrino transitions, giving rise to a, in principle measurable, difference in the overall probability since the CP conserving contributions are the same..

The part of the probability (3) arising from CP violation is $2J\xi$ where

$$\xi = \sum_{i=1}^3 \sin((m_{\nu_k}^2 - m_{\nu_j}^2)L/2E) \quad (4)$$

This sum of sine functions (the sum of whose arguments is zero) may readily be transformed to

$$\xi = 4 \sin(x_d) \sin(y_d) \sin(x_d + y_d) \quad (5)$$

$$(x_d, y_d) = (d_1 L/4E, d_2 L/4E) \quad (6)$$

$$d_1 = (m_{\nu_2}^2 - m_{\nu_1}^2), d_2 = (m_{\nu_3}^2 - m_{\nu_2}^2) \quad (7)$$

. Now consider the function

$$\Xi(x, y) \equiv 4 \sin(x) \sin(y) \sin(x + y) \quad (8)$$

where in $\Xi(x, y)$ the arguments x, y are freely varying and not restricted as in ξ . This function Ξ has multiple maxima, minima with values $+3\sqrt{3}/2, -3\sqrt{3}/2$ at arguments say x_m and y_m which are integer multiples of $\pi/3$ (but obviously not spanning all such integer multiples).

Given mass squared differences then $\xi(L/E)$ is a function varying only with L/E and the above maxima and minima cannot generally be attained. However one can distinguish regions of L/E where relatively high values of ξ are attained. These are, naturally, given by values of x_d, y_d near to x_m, y_m points of Ξ . These latter points can be located in the (x, y) plane through the necessary condition that there the first derivatives of Ξ with respect to both x and y should vanish. A simple geometrical picture can be given as follows.

On the x and y positive quartile of the plane construct a square grid with neighbouring grid lines a distance $\pi/3$ apart resulting in a pattern of squares

of side $\pi/3$. All the maximum and minimum points of Ξ are at intersection points of the grid lines and are given by

$$(x_m, y_m) = (1 + 3l, 1 + 3k)\pi/3 \quad (9)$$

$$(x_m, y_m) = (2 + 3l, 2 + 3k)\pi/3 \quad (10)$$

where l and k are any non-negative integers. The points (9) have $\Xi = 3\sqrt{3}/2$ and the points (10) have $\Xi = -3\sqrt{3}/2$.

It is near these special points in the x, y plane that $\xi(L/E)$ (eqn. 4) has numerically large values. Note that for seeking observation of CP violation using the difference between ν and $\bar{\nu}$ transitions it does not matter whether ξ is positive or negative so both maximum and minimum points of Ξ are equally potentially important.

As L/E varies the points (x_d, y_d) (7) trace a straight line in the (x, y) plane starting at $(0, 0)$ and ascending as (L/E) increases. This line of ξ makes a small angle $\arctan(d_1/d_2)$ with the y-axis and passes through the archipelago of special points given by (9,10). Points (x_d, y_d) on the line of ξ which are close to the Ξ special points (9,10) give numerically large values of ξ and the associated values of L/E signify neutrinos whose transitions contain a relatively large CP violating part. To give an idea of how much of the plane has a value of Ξ near maximum or minimum then the value of Ξ near the special points should be evaluated. Let δ be the distance between a near point and the special point which it is near to. Then near a maximum or minimum $\Xi = \pm 3\sqrt{3}/2(1 - \Delta)$ respectively. where $\Delta \leq 2\delta^2$. . Thus within an area limited by $\delta = .2$ (noting that a grid square has sides length $\pi/3$) the value of Ξ is nearly equal to that at the special grid point.

So the structure of Ξ is such that for certain intervals (not large) of L/E the contribution of $J\xi$ to CP violation in neutrino transitions (measured by the difference between ν and $\bar{\nu}$ transitions) is much bigger than an average over larger intervals. Such an interval of L/E is when the line of L/E passes near a peak or trough of Ξ . The example that follows is only illustrative, though by happenstance rather striking. There are, obviously, uncertainties in the prescription due to considerable relative uncertainties in the value (though certainly small) of d_1/d_2 and also in the different plaquette values found in different theories.

Take

$$d_1 = 8.0 \times 10^{-5} eV^2, d_2 = 2.5 \times 10^{-3} eV^2,$$

so that $d_1/d_2 = .032, d_2/d_1 = 31.25$. Consider the grid line $y = 62\pi/3$ (given

by $k = 20$ eqn.10). On this grid line Ξ has a minimum value $-3\sqrt{3}/2$ at $x = 2\pi/3$ and the line of ξ crosses the grid line at $x = (2\pi/3 - .016)$ which means that ξ has almost attained the minimum value, $-3\sqrt{3}/2$, and that $L/4E = 62\pi/3d_2 = 0.248 \times 10^5\pi/3$.

The values of the plaquettes defined from the MNS mixing matrix elements, $U_{\alpha i}$, depend on the phases of these elements. A theory can give the moduli and phases of these elements and the consequent plaquette have the virtue of being obviously invariant under phase redefinitions of the eigenstates ν_α and of the eigenstates ν_i . In the absence of experimental data on the phases of the $U_{\alpha i}$, but the existence of some considerable experimental guidance on the moduli it seems not unreasonable to use, as a specimen for present purposes, one of the theories which gives a matrix of moduli squared resembling that of the tri-bimaximal mixing hypothesis [8]. The particular theory used [9] incorporates the values of the mass squared differences given and used above and has the matrix of MNS modulus squared elements:

$$\begin{bmatrix} .638 & .344 & .017 \\ .260 & .331 & .409 \\ .102 & .325 & .573 \end{bmatrix}, \quad (11)$$

bearing a distinct resemblance to the postulated 'ideal' structure of this matrix in tri-bimaximal mixing [8]: (The theoretical model [9] produces the MNS matrix using the normal hierarchy.)

As previously noted $\Im\Pi_{\alpha i} = J_{lep}$ (for all 9 elements of Π) and the contribution of CP violation to the transition probabilities is given by

$$P_{CP}(\nu_\alpha \rightarrow \nu_\beta) = 2\xi(L/E)J_{lep} \quad (12)$$

for all α not equal to β .

For this particular model $J_{lep} = .01744$ and at the nearly minimum point discussed above

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = P(\nu_e \rightarrow \nu_\mu) = .3470 \quad (13)$$

$$P(\nu_\mu \rightarrow \nu_e) = P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) = .5276 \quad (14)$$

the difference of 0.1806. It should be emphasized that this large difference depends not only on the nearness to a special point, which is a concept independent of the any particular model of the MNS matrix but also on the particular model having a maybe atypically large value of J_{lep} . Naturally this type of experimental comparison may yield some knowledge of J_{lep} .

The numerical value of L/E given above in units ev^{-2} may, on inserting the appropriate dimensionful value of $\hbar c$, be related to the experimental conditions through

$$L/4Eev^{-2} = 1.266L(km)/E(Gev) = 0.248 \times 10^5 \pi/3$$

. Values of L/E of this order may be appropriate for atmospheric muon neutrinos created on the opposite side of the earth to the detector. However the large value of L/E highlights the probable accelerator experiment difficulties of getting near to some of the special grid points of Ξ ; this sharply depends on the precise value of the gradient of the line of ξ .

It is clear that there is at present no reliable prediction of detailed experimental results; most importantly because the precise slope of the line of ξ in the (x, y) plane is uncertain, being the ratio of neutrino mass differences. Rather, any value of the construction is that experiment may give information on its physically significant parameters. Information may of course be obtained by computer evaluation of the transition probabilities (3) for very many multiple parameter choices, diligent attention enabling the construction of cognitive or computer maps.

The author thanks David Sutherland and Colin Froggatt for comments on this note..

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